

Flipped Class with Electronic Book and SageMathCell for Linear Algebra

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Abstract

This article describes the authors' teaching experience in flipping the class of a basic undergraduate mathematics course Introductory Linear Algebra. We utilize a full featured free electronic textbook, online lecture notes, an intranet learning management system (LMS) i-campus, the video-sharing website YouTube and a Computer Algebra System (CAS) SageMath in our flipped classroom approach. Prior to coming to the class, the students are assigned a portion of reading assignments from the free e-book and to watch recorded video lectures from YouTube that cover the relevant materials. Hence, both out-of-class and in-class sessions are filled with active learning and interactive engagement. The essential part of subject could be fully covered with our free online math calculation tools in a python-based cloud computing environment. This will allow our students to be prepared for new majors (data science, insurance actuaries, big data, and artificial intelligence) that need more qualified college graduates.

1. Introduction

As educators and practitioners, we strive our best in assisting our students in their learning process. This effort will not only help them to retain the knowledge they learn but also to develop their problem-solving and critical thinking skills that will be useful even after they completed a certain course. Adopting some kind of active learning is one such strategy for better instruction and flipping the class is one feature of active learning which has become popular during the past decade. In this article, we will discuss flipped classroom for an introductory course in Linear Algebra by utilizing a full featured free electronic textbook and Computer Algebra System (CAS) *SageMath*.

The term 'flipped classroom' is more popular for K-12 education. It was coined and popularized by Chemistry teachers Jonathan Bergmann and Aaron Sams from Woodland Park High School, Colorado back in 2007 [2, 3]. At the college level, the term 'inverted classroom' is more commonly used. The term was coined by a group of Economics faculty at Miami University, Ohio in 2000 [8]. Although the term might be new, the idea borrows from the centuries-old method of assigning reading materials to students before they come to the class. If students are prepared

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beforehand, in-class time can be dedicated to active learning and higher-order skills, including problem-solving, discussion and critical thinking.

There are a number of studies on the flipped classroom in the teaching and learning of mathematics, covering the primary, secondary and tertiary education. The following provides some examples of the flipped classroom in a number of mathematical subjects: PreCalculus, Algebra and Geometry [5], College Algebra [1, 15, and 20], Calculus [6, 11, 12, 17, and 18], Multivariable Calculus [22], Statistics [21] and Actuarial Science [4].

There exists research of flipped classroom in Linear Algebra and the list is not exhaustive. Several flipped classroom designs which include flipping a single-topic, an entire course and in the form of a series of the workshop has been proposed [19]. Literature has also provided evidence of the flipped classroom's effectiveness in Linear Algebra. For instance, a comparison between the flipped classroom and the traditional lecture can be found in [10], where students' understanding is better in the former. Another study on students' academic performance and perception is given in [13], where students in the flipped classroom achieved superior comprehension and exhibit a positive attitude in terms of enjoyment and confidence. See also [14] and [16] for further supporting evidence of positive students' perspective, participation, interest and self-directed learning skills in the flipped classroom environment.

In this article, we share our teaching experience in flipping the class for an introductory course in Linear Algebra and attempt to fill the gap of flipped classroom study in this area. There are a number of factors that make this study unique and interesting, i.e. utilization of a free electronic textbook, adaptation of CAS *SageMath* and incorporation of the learning management system (LMS) *i-campus* in teaching and learning.

The organization of this article is given as follows. After this Introduction, Section 2 explains the free electronic textbook and online lecture notes. Section 3 provides features of the flipped classroom in an Introductory Linear Algebra course. This includes video recordings, CAS *SageMath* and some flipped classroom activities. Section 4 provides conclusion to our study.

2. Introductory Linear Algebra

At Sungkyunkwan University (SKKU), Linear Algebra (course code GEDB003) is a three-credit introductory math course offered as a part of Basic Science and Mathematics modules. Out-of-class communication between instructors and students is conducted using an LMS *i-campus*. This LMS has features which are similar to other well-established LMS, such as *Moodle*, *Edmodo*, or *Blackboard Learn*. Instructors can write announcements, upload course materials, assign tasks and initiate open discussion. Students can do the latter as well as sending messages to their instructor. An excellent feature of *i-campus* is the possibility to access and to watch video recordings uploaded earlier.

2.1 Free electronic book

A new model of interactive digital mathematics textbook has been introduced [9]. The subject covers a number of mathematics courses, including Calculus, Linear Algebra, Differential Equations, Probability and Statistics, and Engineering Mathematics. In particular, they discussed a digital textbook on Linear Algebra and focus on their experience in using digital contents and interactive laboratories for developing a new model for a digital textbook in Linear Algebra. They also include some appraisal from the readers who feel that the digital textbook is very useful for them, staff employed by the Bank of Korea and a faculty member from University of California at Berkeley in the US.

We adopt a textbook developed by our own faculty and the electronic version of this book is freely available from the web address <http://ibook.skku.edu/Viewer/LA-Text-Eng> and the Portable Document Format (PDF) file can be downloaded from the above address. It is accessible to anyone

and the students can download and use it free of charge and thus we are promoting the utilization of electronic book (e-book).

The front covers of textbooks are shown in Figure 1. This Linear Algebra e-book contains ten chapters, and the title of each chapter is given in the ascending order as follows: Vectors, System of Linear Equations, Matrix and matrix algebra, Determinant, Matrix Models, Linear Transformations, Dimension and Subspaces, Diagonalization, General Vector Spaces and Jordan Canonical Form. The appendix of the e-book contains sample exam questions with their solutions and some worked out exercises. These exercises have been solved, revised and finalized by the students and the final check is conducted by the main author of the e-book.

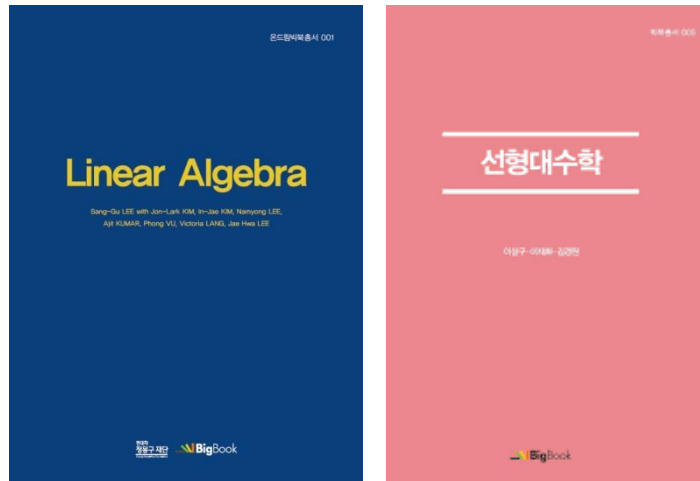


Figure 1: The covers of the adopted textbook 'Bigbook Linear Algebra'. The corresponding English and Korean versions are shown on the left-hand and the right-hand sides, respectively.

The textbook contains many worked examples that will enhance students' understanding and learning of Linear Algebra. There are many interesting reading materials introduced in Chapter 5 of our free electronic textbook, including matrix models in real-life problems such as Lights Out Game, Google matrix, PageRank algorithm, etc. These could motivate students' interest. Some numerical linear algebra knowledge were also introduced, including Power method, inverse power method, Rayleigh Quotient Iteration method, and QR method, which are very useful in practice. It also contains embedded interactive materials with the CAS *SageMath*. Every section of the book includes links to recorded video lectures on *YouTube* and practice web pages. *SageMath* codes are included in the textbook and thus the students can practice using a computer or a mobile device. They can simply copy and paste the commands and the computer code from the e-book to a *SageMath* worksheet or *SageMath* cell server and modify them accordingly to enhance their understanding of the material. For instance, by playing with a larger size of a matrix or by changing some figures from simple ones to more complicated figures, students are able to perceive the usefulness of the CAS and the aid that it provides.

2.2 Online lecture notes

In addition to the Linear Algebra textbook in digital format, recently one of the authors has also developed online lecture notes. These notes are accessible at <http://matrix.skku.ac.kr/LA/>. As can be seen on this website, the lecture notes are arranged according to each chapter of the e-book, and there are also ten chapters in total. Whenever possible, each example is accompanied by *SageMath* code where there is a link to the corresponding web page. We recommend using the Chrome web browser for viewing. This is very useful for a class demonstration using *SageMath* as the instructor can simply

press the ‘Evaluate’ button and the computational result will be displayed accordingly. This action saves time in comparison to copying and pasting the computational codes to a separate cell server.

Furthermore, the *YouTube* video links of the recorded in-class lectures are also displayed below the URL links of the lecture notes. These video links are arranged according to the section number and related topic systematically. This allows the students to access and to watch the recorded video lectures on the relevant topic before they come to the class. Hence, providing the links which are accessible to the students supports the structure of flipped classroom where the students can gain basic information outside the classroom by watching recorded video lectures. In-class time can thus be dedicated to more rigorous problem-solving activities, discussion and feedback.

3. Flipped classroom in Introductory Linear Algebra

3.1 Video recordings

As mentioned previously, one feature of active learning is adopting and implementing flipped classroom approach in our course. In order to make the flipped classroom successful, the students must prepare prior coming to the class and they must participate actively in in-class activities. For the former, assigned tasks include some reading assignments or watching recorded video lectures. In order to make sure that they come prepared, a small assignment can be embedded into the task and the mark from this assignment is to be included in the final grade calculation.

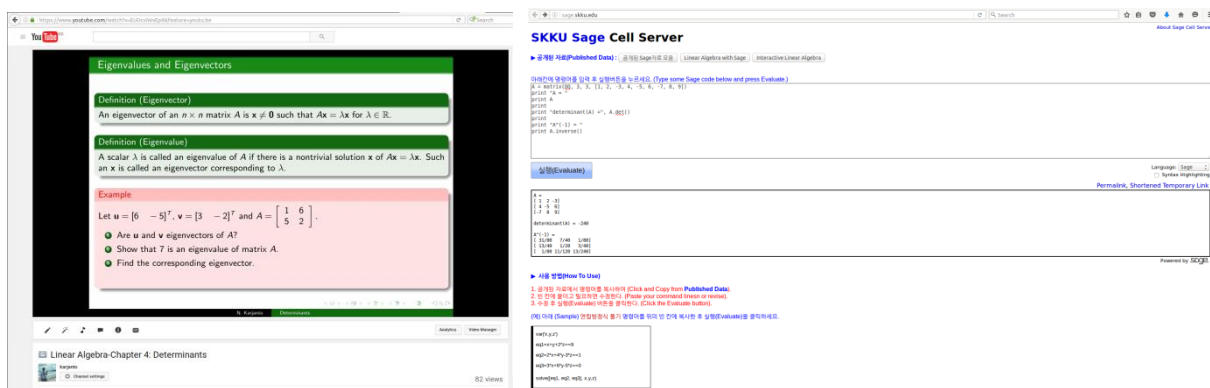


Figure 2: (Left) A screenshot from a *YouTube* page which shows screencast video material of Linear Algebra on the determinant, eigenvalue and eigenvector. (Right) A screenshot of the SKKU’s SageMath cell server with an example of calculating a determinant of a 3×3 matrix and the inverse of a matrix.

When it comes to selecting video lectures to be assigned to the students, we have many options. We can record our own lectures, upload them to a video-sharing website and send the URL link to the students or we can use videos from other instructors available online. We have decided to choose the former. For the latter, one can access them from the video-sharing website *YouTube*. Uploading video recordings online will be useful not only in helping the students to prepare materials but also beneficial in case they are interested in viewing them again at a later time. In addition, a personalized learning resource *Khan Academy* has more than 2400 free instructional videos on many different subjects. Depending on the materials, the duration of the videos may range from short (around five minutes) to long (around one hour). The screenshot of the video recordings is shown Figures 2 and 3. The left panel of Figure 2 shows a screenshot from a *YouTube* page showing a screencast video material of Linear Algebra on the determinant, eigenvalue and eigenvector. The right panel of Figure 2 shows a screenshot of the SKKU’s *SageMath* cell server with an example of calculating a 3×3 matrix’s determinant and a matrix’s inverse. The left panel of Figure 3 shows a screenshot from a *YouTube* page which shows screencast video material of Linear Algebra on

subspace. The right panel of Figure 3 displays a screenshot of the Analytics data page from *YouTube*. From the Analytical data provided by *YouTube*, we observe that the most popular videos that the students watch are the topics on Subspace, Linear Transformation and Finding the Jordan Canonical Form, where each video has been viewed for more than 600 times.

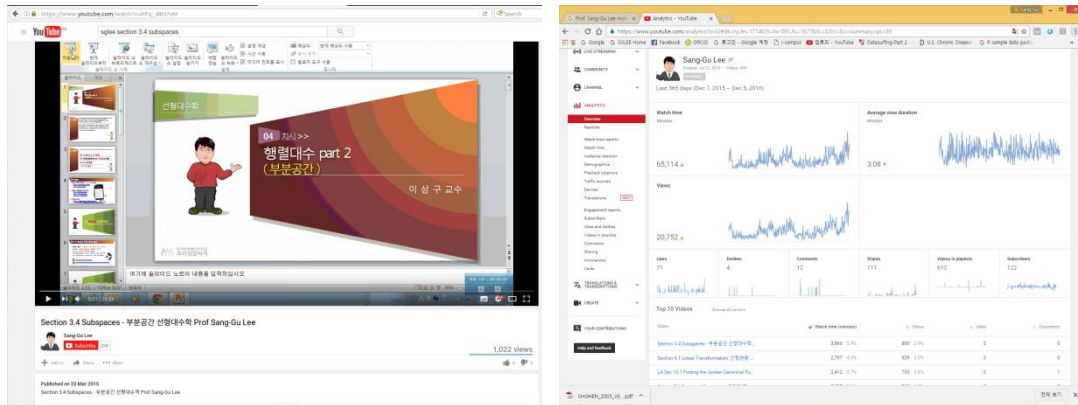


Figure 3: (Left) A screenshot from a YouTube page which shows screencast video material of Linear Algebra on subspace. (Right) A screenshot of the Analytics page from YouTube.

3.2 Computer Algebra System SageMath

SageMath is a computer algebra system (CAS) and it is also a free open-source mathematics software. According to its official website, the mission of disseminating *SageMath* is to create a viable free open source alternative to *Magma*, *Maple*, *Mathematica* and *Matlab*. See <http://www.sagemath.org/>. It is distributed under the terms of the GNU General Public License version 2 + . It can be utilized to assist computations in Calculus, Linear Algebra, Combinatorics, Number Theory and Numerical Methods.

In this course of Introductory Linear Algebra, *SageMath* is utilized in both in-class and out-of-class activities. In-class activities include demonstrations of solving Linear Algebra problems using *SageMath* and problem-solving sessions for the students to try the CAS by themselves. Out-of-class activities include assignments for students to solve Linear Algebra problems using the CAS. For the former, the instructor and the students visit an online ‘single cell’ version of *SageMath* accessible at <https://sagecell.sagemath.org/> and perform a computation there. The cloud computing platform is available from our own university’s cell server and is accessible at <http://sage.skku.edu/>. The following subsection provides examples of flipped classroom activities by utilizing CAS *SageMath*.

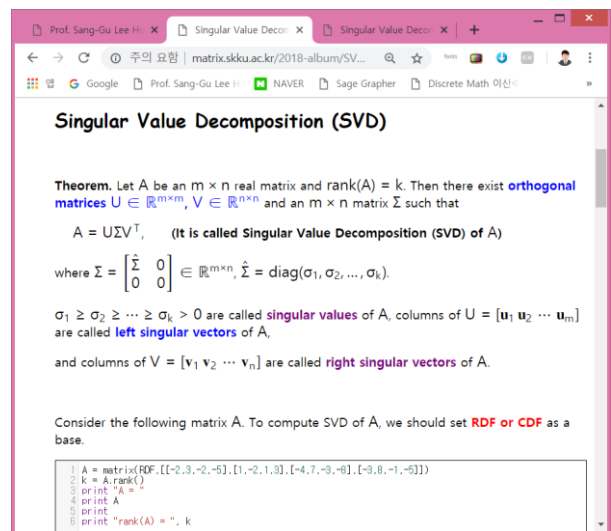
3.3 Flipped classroom activities

Prior coming to the class, students are assigned to read a portion of the textbook on a relevant topic covering a particular week’s material. They are also requested to watch video lectures on the relevant material, usually from *YouTube*. The tasks are communicated via the LMS *i-campus*. After a brief summary of basic definitions and theorems, a typical class time is dedicated to a problem-solving session and a workshop on the use of technology. Earlier in the semester, we demonstrate some computational examples using CAS *SageMath* to the students and they are also invited to attempt solving problems using the software. The purpose is to help them to get more familiar with the CAS. We also select relatively simple examples with small matrix size so that the students can also confirm the computational results using hand computation. Later in the semester, they conduct the demonstration less frequently as the students have obtained a level of familiarity and they can figure out the syntax.

During the problem-solving session, the students can work individually or in a team that they form by themselves. On moving toward creating a collaborative learning environment, the latter is usually encouraged. We navigate around the classroom to monitor the students working on assigned problems. This provides an opportunity not only for probing students' progress on the tasks and their understanding of Linear Algebra concepts but also assists them whenever they encounter difficulties. We observe that students are more open and less hesitant to ask questions during this session in comparison with the end-of-lecture session, where they are also given a chance to ask questions.

For a weaker group of students, we can personalize our approach by guiding them in solving problems with a slower step-by-step inquiry and explanation. They are also encouraged to practice more problems outside the problem-solving sessions and during out-of-class activities. For an academically stronger group of students and those who have a keen interest in mathematics, we provide them with more challenging and computationally demanding problems. We also encourage them to solve problems not only by hand computation but also using the CAS and ask them to confirm the results. In the case of some discrepancy, we inquire them whether there exists potentially some computational errors or the results are simply a matter of different representations of the solution. This may lead to further discussion in Linear Algebra, including conceptual one. Hence, in the flipped classroom, we tailor our pedagogical approach in satisfying the needs of students with diverse academic backgrounds. In either case, our role as instructors is to 'guide on the side' rather than to be a 'sage on the stage' [7].

The following provides some examples of in-class and out-of-class problem-solving activities with the use of CAS *SageMath*. We start with the smaller size of matrices and then we gradually increase to a larger size of matrices. The first example is to ask the students to find a singular value decomposition (SVD) of 2×2 or 3×3 matrices, first using pen and paper, and then using the CAS. For some special cases, larger size of matrices can also be found by hand calculation, but CAS *SageMath* gives a faster result, for any size of matrices. The following gives a CAS *SageMath* code for finding a SVD of a 6×4 matrix randomly generated by using our SVD lab, which was developed and used for student's activities in the following URL address <http://matrix.skku.ac.kr/2018-album/SVD1.html>.



```
A = random_matrix(RDF, 6, 4) # 6 by 4 random matrix A
U,S,V = A.SVD() # Find SVD of A. A = U*S*V'
print "U = "
print U
print
print "S = "
print S
print
print "V = "
print V
```

The computational result is displayed as follows.

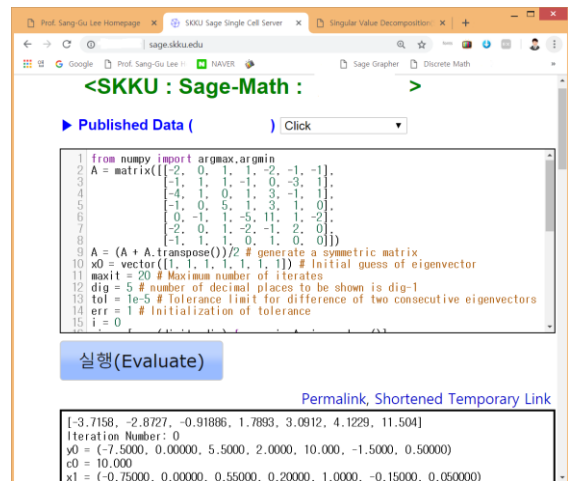
```
U =
[ -0.330940942074  0.300931413542  0.375290296466 -0.625706610501 -0.41929970804 -0.302910156638]
[  0.637349062067  0.265755394823 -0.43454193854 -0.326050050514  0.119335697669 -0.462367713705]
[  0.243502248321 -0.720748885241  0.130274879149  0.117514262369 -0.474630505748 -0.406414119467]
```

```
[ 0.184900000084  0.476274254799 -0.141748863003  0.436415925791 -0.706336976979  0.171788528982]
[ -0.613855515548 -0.0849729042376 -0.712676583328  0.0727344132567 -0.103791775474 -0.303298650879]
[ 0.118170711281 -0.29202369536 -0.354059741151 -0.540947174056 -0.273839316768  0.638582592553]

S =
[ 1.85859983624      0.0      0.0      0.0]
[      0.0  1.6646153427      0.0      0.0]
[      0.0      0.0  0.565896159427      0.0]
[      0.0      0.0      0.0  0.123936917311]
[      0.0      0.0      0.0      0.0]
[      0.0      0.0      0.0      0.0]

V =
[ 0.546728249985 -0.340596162244  0.13744451879  0.752456961685]
[ -0.52997266672  0.107195714593  0.789896747109  0.289311562521]
[ -0.440939604957 -0.882138066294 -0.157782946504 -0.0500923004841]
[ -0.475173082127  0.307141278621 -0.576434449832  0.589574509316]
```

Another example of out-of-class activities is an assignment for the students to adopt CAS *SageMath* or any other CAS they are familiar with to work on finding a dominant eigenvalue and the eigenvector corresponding to that particular dominant eigenvalue of a 7×7 symmetric matrix A with an initial condition x_0 generated by students themselves. The students need to solve this problem numerically by implementing the ‘power method’ algorithm and they were required to provide a minimum of ten iterations. A dominant eigenvalue of a matrix is the largest eigenvalue in the absolute value sense and the dominant eigenvector is the eigenvector corresponding to the dominant value. An example similar to this problem can be found in the free e-book, including the *SageMath* code and the corresponding computational result. A common mistake is that the students calculate eigenvalues and the corresponding eigenvectors of the matrix analytically, instead of numerically, using CAS *SageMath* and other CAS. Another common mistake that was observed is that although the students attempt to use power method, they did not provide a minimum of ten iterations. The following gives a CAS *SageMath* code for finding the dominant eigenvalue and the eigenvector corresponding to this dominant eigenvalue for a matrix given by a student. All can check how the code works in <http://sage.skku.edu/>



```
from numpy import argmax, argmin
A = matrix([[ -2,  0,  1,  1, -2, -1, -1],
            [-1,  1,  1, -1,  0, -3,  1],
            [-4,  1,  0,  1,  3, -1,  1],
            [-1,  0,  5,  1,  3,  1,  0],
            [ 0, -1,  1, -5, 11,  1, -2],
            [-2,  0,  1, -2, -1,  2,  0],
            [-1,  1,  1,  0,  1,  0,  0]])
A = (A + A.transpose())/2 # generate a symmetric matrix
x0 = vector([1, 1, 1, 1, 1, 1, 1]) # Initial guess of eigenvector
maxit = 20 # Maximum number of iterates
dig = 5 # number of decimal places to be shown is dig-1
tol = 1e-5 # Tolerance limit for difference of two consecutive eigenvectors
err = 1 # Initialization of tolerance
i = 0
eig = [ev.n(digits=dig) for ev in A.eigenvalues()]
print eig # compute eigenvalues using built-in SageMath command
```

```
while(i <= maxit and err >= tol):
    y0 = A*x0
    ymod = y0.apply_map(abs)
    imax = argmax(ymod)
    c1 = y0[imax]
    x1 = y0/c1
    err = norm(x0-x1)
    i = i + 1
    x0 = x1
    print "Iteration Number:", i-1
    print "y"+str(i-1)+" =", y0.n(digits=dig)
    print "c"+str(i-1)+" =", c1.n(digits=dig)
    print "x"+str(i)+" =", x0.n(digits=dig)
    print
```

The computational result is displayed as follows.

```
[-3.7158, -2.8727, -0.91886, 1.7893, 3.0912, 4.1229, 11.504]
Iteration Number: 0
y0 = (-7.5000, 0.00000, 5.5000, 2.0000, 10.000, -1.5000, 0.50000)
c0 = 10.000
x1 = (-0.75000, 0.00000, 0.55000, 0.20000, 1.0000, -0.15000, 0.050000)

Iteration Number: 1
y1 = (-0.15000, 0.60000, 3.7750, 0.92500, 12.625, 0.72500, 0.80000)
c1 = 12.625
x2 = (-0.011881, 0.047525, 0.29901, 0.073267, 1.0000, 0.057426, 0.063366)

...

Iteration Number: 11
y11 = (-1.0631, -0.34769, 1.9456, -0.53489, 11.504, 0.25079, -0.26868)
c11 = 11.504
x12 = (-0.092411, -0.030223, 0.16912, -0.046497, 1.0000, 0.021800, -0.023355)

Iteration Number: 12
y12 = (-1.0631, -0.34770, 1.9455, -0.53491, 11.504, 0.25080, -0.26869)
c12 = 11.504
x13 = (-0.092411, -0.030225, 0.16912, -0.046498, 1.0000, 0.021801, -0.023356)
```

From this result, we observe that the dominant eigenvalue is $c_{12} = 11.504$ and the eigenvector corresponding to the dominant eigenvalue is $x_{13} = (-0.092411, -0.030225, 0.16912, -0.046498, 1.0000, 0.021801, -0.023356)$.

Students solved these problems as they are, or they revise and modify them, and upload their own solutions for three to four problems every week in the LMS QnA. At the same time, students do review and correct those solutions that other students have generated, or encourage them to ask for details of the solution that were not easily comprehended by themselves. At the same time, the participant's record is tagged so that the discussion process can be seen as shown in the following example.

- 2018. 04. 06. Question/Solution by Oh** (New problem and the first solution)
- 2018. 04. 06. Answer/Revised solution by Lee** (revision)
- 2018. 04. 07. Finalized by Oh** (Complete the revision)
- 2018. 04. 08. Finalized OK by Prof. *** or TA *** (Prof/TA confirms and grades)

- better and by the end of this semester I hope we will have acquired many skills other than just academic ones. Once again thanks to everyone for making this experience way more effective.
6. I think these classes were useful for me as a student of Software Engineering department.
 7. To provide a recap, this is my first experience in studying under that kind of system, hence it was slightly difficult to adapt. However, I am happy that I have an opportunity to experience it, since this is the first class I have ever had where every single student is involved in the process of learning and activity of everyone is important for every student as well since we help each other to understand the concept of the class quicker by solving or revising and finalizing others' solutions. I like QnA section because it became a kind of medium where students can have a discussion on any problem and just simply make any inquiry. This system of learning motivates me to use various materials, such as articles on the Internet, *YouTube* videos, books and QnA responses, in the process of learning and to study before the lecture to fully grasp the idea of the lecture.
 8. Firstly, this lesson gives us an incentive to study hard, by comparing our work with the work of other students. Secondly, the lesson develops our logical thinking abilities and expands our horizons. This lesson (PBL) allows us to learn non-stop using not only the material we listened at the lecture, but also our professor's website (lecture note/*SageMath* codes.), various textbooks and videos (videos of our professor) and several of another educational materials. Finally, this lesson improves our communication skills. Solving problems, and also revising and finalizing homework problems of other students, we are sharing non-stop with each other various methods for solving mathematical tasks, and also through this process we give each other various tips for learning discrete mathematics more effectively. Therefore, I think that this PBL (active learning) technique is very good and effective for learning.

The final grade can be given based on PBL reports, on /offline attendance, QnA participation frequency, quiz score, midterm offline exam, final exam etc. Figure 5 shows one example of an evaluation sheet.

Name	Online	on 출석	Q	A	Q&A	off 출석	Quiz/30	Quiz	MidTerm	Exam	PBL	midSUM	Grade
***	120	5	6	14	7	5	26	17.33	90	45	10	89.33	A+
***	116	5	17	13	9	5	20	13.33	90	45	10	87.33	A+
***	109	5	10	16	9	5	21	14.00	89	44.5	8	85.50	A
***	109	5	3	11	7	5	18	12.00	82	41	10	80.00	A
***	94	4	6	5	6	5	14	9.33	88	44	9	77.33	B+
***	104	5	9	2	7	5	17	11.33	75	37.5	7	72.83	B+
***	108	5	8	17	8	5	14	9.33	67	33.5	9	69.83	B
***	98	4	1	4	4	4	18	12.00	73	36.5	5	65.50	B
***	95	4	2	1	3	4	16	10.67	52	26	9	56.67	C+
***	16	2	2	0	1	2	4	2.67	47	23.5	3	34.17	D+
***	0	0	0	0	0	1	1	0.67	0	0	1	2.67	F

Figure 5: Evaluation sheet

Now this flipped class improved students' ability to practice and solve Linear Algebra problems for larger-sized matrices without any difficulties. Students try to modify and create some new codes for solving more difficult problems by applying pre-coded *SageMath* cells. This is possible only when using free non-commercial software. As mentioned above, many students became confident with their improved mathematical and computational skills. According to statistics, each class started with 60-70 students at first. Among them, about 70% students survived after the whole semester work. More than 15% students made remarkable progress. As a result, the main author received a 2018 teaching award.

This model was much easier for teachers to motivate and evaluate student's performance, but it also allowed students to develop critical thinking skills while describing and presenting their learning and problem-solving processes together with classmates.

4. Conclusion

In this article, we have discussed our experience in flipping the classroom of an Introductory Linear Algebra course at SKKU. A new feature of our teaching includes developing a new, free electronic textbook that can be accessible to everyone in the world and disseminating online lecture notes on a single web page that can be used by both students and instructors alike. We have embedded the use of technology in our teaching, in particular exploring the power of open access and free CAS *SageMath* for computational purposes. For communication between instructors and students outside the classroom, we utilize an intranet LMS *i-campus*. This platform is also used to provide additional teaching materials, to send announcements and to foster discussion in an online forum.

We have also created recorded video lectures, either in a screencast format or in the form of real live classroom teaching. These videos are accessible from the video-sharing platform *YouTube* and the students can watch them primarily not only before they come to the class sessions, but also after the class if they wish to review the lectures again at their own pace. The students are expected to gather basic information on definitions, properties, theorems, methods and techniques in Linear Algebra prior the class time, by either reading assignments, or watching recorded video lectures. We dedicated class time to activities that constitute active learning, including problem-solving, question and answer, discussion and providing feedback. Overall, we have gleaned valuable lessons in flipping the class so that we can improve and implement a better strategy in the flipped classroom pedagogy for future courses we are going to teach.

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5. References

- [1] Acelajado, M.J., Flipped teaching approach in College Algebra: Cognitive and non-cognitive gains, In *Proceedings of the 13th International Congress on Mathematical Education*, Gabriele Kaiser (Ed.). pp. 697-698, 2017.
- [2] Bergmann, J. and Sams, A., *Flip your classroom: Reach every student in every class every day*, International Society for Technology in Education, 2012.
- [3] Bergmann, J. and Sams, A., *Flipped learning: Gateway to student engagement*, International Society for Technology in Education, 2014.
- [4] Butt, A., Student views on the use of a flipped classroom approach: Evidence from Australia, *Business Education & Accreditation*, 6(1), pp. 33, 2014.
- [5] Fulton, K., Upside Down and Inside Out: Flip Your Classroom to Improve Student Learning, *Learning & Leading with Technology*, 39(8), pp. 12-17, 2012.
- [6] Jungic, V., Kaur, H., Mulholland, J. and Xin, C., On flipping the classroom in large first year Calculus courses, *International Journal of Mathematics Education*, 46(4), pp. 508-520, 2015.
- [7] King, A., From sage on the stage to guide on the side, *College Teaching*, 41(1), pp. 30-35, 1993.

- [8] Lage, M.J., Platt, G.J. and Treglia, M., Inverting the classroom: A gateway to creating an inclusive learning environment, *The Journal of Economic Education*, 31(1), pp. 30-43, 2000.
- [9] Lee, S.-G., Lee, J.H. and Park, K.-E., Development and usage of interactive digital Linear Algebra textbook, *Journal of the Korean Society of Mathematical Education Series E: Communications of Mathematical Education*, 31(3), pp. 123-138, 2017.
- [10] Love, B., Hodge, A., Grandgenett, N. and Swift, A.W., Student learning and perceptions in a flipped Linear Algebra course, *International Journal of Mathematical Education in Science and Technology*, 45(3), pp. 317–324, 2014.
- [11] Maciejewski, W., Flipping the calculus classroom: an evaluative study, *Teaching Mathematics and its Applications: An International Journal of the IMA*, 35(4), pp. 187-201, 2016.
- [12] McGivney-Burelle, J. and Xue, F., Flipping Calculus, *Primus*, 23(5), pp. 477-486, 2013.
- [13] Murphy, J., Chang, J.-M. and Suaray, K., Student performance and attitudes in a collaborative and flipped Linear Algebra course, *International Journal of Mathematical Education in Science and Technology*, 47(5), pp. 653-673, 2016.
- [14] Novak, J., Kensington-Miller, B. and Evans, T., Flip or flop? Students' perspectives of a flipped lecture in mathematics, *International Journal of Mathematical Education in Science and Technology*, 48(5), pp. 647-658, 2017.
- [15] Overmyer, G.R., *The flipped classroom model for College Algebra: Effects on student achievement*, Ph.D. Dissertation, Colorado State University, Fort Collins, Colorado. 2014.
- [16] Park, K.-E. and Lee, S.-G., Flipped Learning teaching model design and application for the University's 'Linear Algebra', *Journal of the Korean Society of Mathematical Education Series E: Communications of Mathematical Education*, 30(1), pp. 1-2, 2016.
- [17] Sahin, A., Cavlazoglu, B. and Zeytuncu, Y.E., Flipping a college calculus course: A Case study, *Journal of Educational Technology & Society*, 18(3), pp. 142-152, 2015.
- [18] Sonnert, G., Sadler, P.M., Sadler, S.M. and Bressoud, D.M., The impact of instructor pedagogy on college calculus students' attitude toward mathematics, *International Journal of Mathematical Education in Science and Technology*, 46(3), pp. 370-387, 2015.
- [19] Talbert, R., Inverting the Linear Algebra classroom, *Primus*, 24(5), pp. 361-374, 2014.
- [20] Sickle, J.V., Adventures in flipping College Algebra, *Primus*, 25(8), pp. 600-613, 2015.
- [21] Wilson, S.G., The flipped class: A method to address the challenges of an undergraduate Statistics course, *Teaching of Psychology*, 40(3), pp. 193-199, 2013.
- [22] Ziegelmeier, L.B. and Topaz, C.M., Flipped Calculus: A Study of Student Performance and Perceptions, *Primus*, 25(9-10), pp. 847–860, 2015.